Harnessing Formative and Summative Assessments to Promote Mathematical Understanding and Proficiency

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Abstract

Whenever attempts are made to diagnose the cause of mathematics underachievement, sometimes the real problem is missed leading to the claims that engender unfortunate dichotomy other than harmony between complementary concepts. Mathematics educators must work to ensure and maintain the realization that mathematics is a connected enterprise. Against this backdrop, prompting a feud between for example, basic skills and conceptual understanding or a disagreement between formative assessment and summative assessment as competing issues does not promote effective mathematics learning. Assessment plays formidable role in mathematics education hence both formative and summative assessments should be aligned in a meaningful way to effect success in mathematics understanding and proficiency. In this article the relative merits of formative and summative have been discussed and there is little doubt that the two forms of assessment are more in agreement than conflict. Formative assessments can be conceived as micro summative assessment and for that matter they have identical or complementary objectives of determining mathematics learning outcomes. Even though the two forms of assessment are administered at different time points, they can be characterized as mutually reinforcing.

Introduction

Mathematics education is about the curriculum, teaching, learning, and assessment. In this regard, the National Council of Teachers of Mathematics (NCTM, 2000) standards for K-12 mathematics have stipulated that mathematics education will ensure "solid mathematics curricula, competent and knowledgeable teachers who can integrate assessment with instruction" (p. 3). Furthermore NCTM advocates teaching and learning performance in a technology-rich environment. The NCTM standards (content and process) have outlined the requisite mathematics knowledge, understanding, and skills expected of learners in order to be proficient in mathematics.

Whereas NCTM (2000) refers to problem-solving, reasoning and proof, communication, representation, and connections as the processes for mathematics learning, RAND (2003) emphasizes mathematical practice that "focuses on the mathematical know-how, beyond content knowledge, that constitutes expertise in learning and using mathematics" (p. xviii). Of much relevance is the concern that, teachers will be able to develop in their students the required proficiency and ensure its prevalence using class assessments. This essay focuses on the complementarity of formative and summative assessments in mathematics education. The paper offers an anatomical critique of summative and formative assessments dichotomy, while providing symmetrical reflections that teachers could possibly exploit to effect mathematical understanding and proficiency among their students. The author concludes that in the field of mathematics, while summative and formative assessments test at different times students' understanding and comprehension of curriculum, both types of assessments are mutually reinforcing.

Mathematical Comprehension

Assessing mathematical proficiency among students cannot be dissociated from mathematical comprehension or understanding. Parallel views have been expressed in the literature regarding mathematical understanding and how it unfolds. NRC (2001) has indicated that mathematical concepts, operations, and relations are mutually dependent and collectively intertwined with the other aspects namely, *procedural fluency, strategic competence, adaptive reasoning,* and *productive disposition* (NRC, 2001).

Kieren and his colleague Pirie have modeled the growth of individual mathematical understanding in concept building as the expression of *primitive knowing* through *image making, image having, property noticing, formalizing, observing, structuring and* inventizing (Kieren, 1994). The basic idea of this model is that the learner brings to any learning situation some previous knowledge, which should be built upon for the understanding and learning of the new concept.

Relatively, the modes of understanding in Kieren's (1994) model are "found to be a dynamic non-linear process" (p. 214), or as a "kind of dynamically woven pathway" (p. 218). Kieren (1994) asserts that mathematical understanding activity is not disjoint or detached but changes on a continuing basis. From the episodes of Kara and Tanya (Kieren, 1994) regarding their mathematical understanding of fractions, the authors concluded that the occurrence of understanding could neither be associated with "types or acquisitions" (p. 214) nor "pretest/post-test differences" (p. 218), but as a "sequence of action events."

In a similar opinion Cain, Carry, and Lamb (1985) have expressed in their model *conceptual mathematics* that emphasizes "understanding of quantitative and spatial relationships and concepts" (p. 24). They state in their model that "intuition, graphic representations, and numerous examples from the experiences of students are used to establish, broaden, and reinforce concepts" (p. 24). Thus, all three opinions share consensus on integrated processes for promoting mathematical understanding. Critical to students' mathematical understanding and consequent proficiency depends on the pedagogical expertise that is brought to bear on the learning situation.

Pedagogy

A plethora of studies have shown that , the major factor, which influences student learning, is the teacher (Roueche & Roueche, 1995; NCTM, 2000). The teacher must have positive attitudes both towards mathematics and the use of resources such as manipulatives and hand-held devices (for example calculators) to make mathematics interesting and meaningful to students (Schwartz, 2000). According to Darling-Hammond (1993), the complex nature of society with its challenging social situations would not accommodate educational system that packages instructions to students but rather one which affords the student the advantage of higher order learning; a system which does not require teachers only to cover a curriculum but also to ensure that in the instructional process, students are involved in their own knowledge construction and the development of their talents in a variety of ways.

A notable school of thought finds expression in the claim that the foundations of students' mathematical education within the context of United States largely concentrate on procedural knowledge and skills thereby compromising conceptual understanding (NRC, 2001). In China, this foundation is referred to as the "two basics" and the establishment of a good foundation should be taken up by mathematics education during elementary and secondary years (Zhang, Li, & Tang, 2004). To be able to accomplish this task would require effective teaching which is a function of teachers knowledge, mathematical content, and students' individual engagement on mathematical tasks (NRC, 2001).

Consistent with NRC (2001), RAND Mathematics Study Panel (2003) and NCTM (2000) regarding *adaptive reasoning, mathematical practices* and *communication standard* respectively, Foley (2008) opines, "mathematical thinking, mathematical proficiency, and mathematical practices are closely linked to and dependent on language and communication." In this light teachers' efforts at assessing mathematical learning should incorporate mathematics language development, because it underscores the realization of both mathematical comprehension and proficiency.

Formative versus Summative Assessment

In spite of the consensus on the goals of mathematics education, this field of educational pursuit continues to deal with unfortunate dichotomies that have not escaped critique of mathematics educators. In juxtaposition to the bogus dichotomy between basic skills and conceptual understanding as discussed by Wu (1999), an apparently incontrovertible parallelism is the controversy between formative and summative assessments in mathematics education.

Assessment options open to the teacher could be categorized into *external assessment*, *classroom assessment,* and *alternative assessment.* External assessment such as national, state, or district assessment, conveys to students, teachers, parents, and the public what is deemed important for instruction and learning in the classroom. External assessment is synonymous with summative assessment. On the other hand the daily students' classroom assessment or *formative assessment* by the teacher conveys to the students and parents what the school and teacher consider valuable in student performance (North Central Regional Educational Laboratory [NCREL], 2008). In practice we should expect a concurrence in the outcomes of what both external and formative assessments seek to establish. Lack of consistency in summative and formative assessment outcomes could be interpreted as a discrepancy between the two complementary forms of assessment (Black, Harrison, Lee, Marshall, & William, 2003).

Formative Assessment

Undoubtedly, summative assessment would not be a viable route for assessing mathematical understanding in a sequence of action events. Rather, the use of formative assessment would help teachers create instructional environment that promotes the growth of understanding as a process, and observe students' understanding as it grows out of the process (Black, Harrison, Lee, Marshall, & William, 2003; NRC, 2001; Kieren, 1994).

The far-reaching implication is that teachers should place formative assessment of mathematical understanding at the heart of instruction using questioning, feedback, self- and peer-assessment, and formative use of summative assessment (Black, Harrison, Lee, Marshall, & William, 2003). The authors have expressed preference for ‘comments only’ feedback assessment in the sense that grades tend to de-motivate under achievers and lower their self-esteem. Furthermore, they argue that peer- and self-assessment allow students to effectively monitor their learning and collaborate with teachers in class management.

As students exercise personal agency, the importance of language in teaching, learning, understanding and communication of mathematics cannot be overemphasized (Ríordáin & O’Donoghue, 2008). According to Roubíček (2008), when students invent their own representations which deviate from acceptable mathematical laws or procedures, “it is necessary to intervene in due time for the reason that postponed rectification or reeducation is an exacting and lengthy process.” (p. 7).

Asking appropriate questions could tease out mathematical understanding. According to Kieren (1994), such understanding “can be observed in terms of the number of epistemological obstacles faced”. Using conversations in formative assessment could also produce desirable results. In this instance, students come to a deeper understanding as they “correct their errors, and internalize their newly acquired knowledge” (Vanderhye & Zmijewski, 2007/2008, p. 261). Vanderhye and Zmijewski (2007/2008) have argued that these errors (or obstacles) are the raw materials that inform teachers’ instructional decisions.

The effective use of questions in assessing students’ understanding should be considered within the context of the students’ wait time as they participate in question-and-answer discussions. To increase participation, students should be allowed to share ideas among themselves before finally asking for contributions from students (Black, Harrison, Lee, Marshall, & William, 2003). The authors emphasize the use of ‘big questions’ that promote open discussions or problem solving tasks that could set the tone of the class lesson. A distinction should be drawn between objective questions that call for ‘Yes/No’, ‘True/False’ answers and subjective questions, which involve extended-response questions. Current formative practice would give preference to the latter because it opens the opportunity for students to discover multiple ways of giving alternative answers. In such a situation students communicate, reason, and represent their ideas thereby developing their mathematics language acquisition critical to mathematical understanding and proficiency.

An important research evidence for the use of formative assessment is in the area of diagnostic assessment that determines the appropriate placement of a child at a point in relation to his or her learning needs using criterion-referenced model. Bergan et. al (1991) (as cited in Black, Harrison, Lee, Marshall, & William, 2003) have concluded that conventional teaching under develops the capacity of children thus illustrating "the embedding of a rigorous formative assessment routine within an innovative programme linked to a criterion-based scheme of diagnostic assessment."

A recent development in formative assessment is the alternative form of assessment. Alternative assessment includes any form of assessment which encourages students to give higher order responses rather than select from a list of possible responses. Alternative assessments are becoming common as a formative practice because they measure the more complex learning goals that we now hold for students and support the instruction necessary for students to achieve these goals (NCREL, 2008).

Finding an alternative assessment has been emphasized as a process for ensuring the mathematical understanding of students. Alternative assessments yield equitable assessment practices through multiple assessment strategies that reveal the strengths of all students.

Golding (2007) exposition on cup-cake quiz provides to a large extent an applicable example of alternative assessment technique. It is consistent with the opinion of McMunn (2000) that, "classroom assessment is an ongoing process through which teachers and students interact to promote greater learning" (p. 6).

Sanchez and Ice (2004) have identified open-ended assessment items capable of bringing out students' "mathematical thinking, reasoning processes, problem-solving and communication skills." In their opinion, the traditional way of asking questions leads to memorization of procedures. However open-ended questions offer a variety of ways of eliciting *what* students know, and *how* they explore the concepts. For example students can routinely solve simultaneous linear equations of two unknowns. However, given the solution set of simultaneous linear equations of two unknowns, students find it difficult to come up with the right equations.

Summative Assessment

Formative assessments are characterized as "frequent informal testing (i.e. as 'micro-summative assessment" (p. 122). Considered as micro-assessment, formative assessment has a lot in common with summative assessment, and justifies the logic of the formative use of summative assessment. Thus summative assessment has an important role to play if it is administered appropriately in instruction and learning.

Siu (2004) has opined that terminal examination (summative assessment) can benefit both the teacher and student. Among the identified benefits, the student can consolidate knowledge, improve his understanding, and plan a study schedule, judge what is important to learn, come out with learning strategies and get motivated from the self-perception of being competent. Similarly, teachers can monitor how the students are progressing and also the effectiveness of their teaching.

McLoed (2007) has summarized the biological similarities between the humans and the animal kingdom that brought to the fore an important area of psychological investigations called behaviorism historically attributed to Ivan Pavlov, John Watson and Skinner in the beginning of and through the 19th century. Through the well-known classical and operant conditioning, the behaviorists applied the processes of stimulus and reinforcement to determine how new behavior is formed. Furthermore, Gredler (2001) opines that the behaviorists' theory is based on observable changes in behavior of the learner in his/her environment. Moreover, the focus of the theory is directed to the repetition of a new behavioral pattern until it becomes automatic.

The behaviorist theory of learning has found considerable application in summative assessment. As a framework, Schwier (1998) introduces the mnemonic device 'ABCD' in an example that implements summative assessment, where the teacher's objective could be:
'The student will be able to complete at least 85% of the questions on the posttest after having completed the lesson or unit in say an algebra lesson.' Schwier (1998) explains that:

- A represents 'Audience', that is, the student;
- B represents 'Behavior', that is, the correct answer;
- C represents 'Condition', that is, after having completed the unit on a posttest; and
- D represents 'Degree', that is, 90% correct.

Terminal examinations are necessary to assess performance at the end of a module, course, or program and this role can well be performed through the administration of summative assessment.

Behavioral objective is usually stated in "specified, quantifiable, terminal behaviors" (Saettler, p. 288, 1990). To develop behavioral objectives a learning task must be broken down through analysis into specific measurable tasks. He explains further that, behavioral theorists are concerned with statement of objectives that are clear, observable, measurable and achievable (COMA). The restriction of the behaviorist approach to observable behavior raised concerns about knowledge or behavior change which is not observable but which could be exhibited at the least opportunity.

Considering the behavioral change expected in the last stage of the ‘ABCD’ device for summative assessment, it would be difficult to rule out the mental processes the learner goes through in accomplishing a task after going through a lesson module. To this end, the cognitive theory is at variance with the behaviorist’s sole emphasis on observable behavior, coupled with the passive posture of the learner in the learning situation (Phillips & Soltis, 2004). Thus, a compelling case is made for formative or alternative assessments that addresses assessment of learning outcomes in the sequence of action events alluded to in previous paragraphs. These forms of assessment work hand in hand.

Usiskin (2012) discusses Singaporean students’ success on international assessments citing the combination of the end of sixth-grade high-stakes test and additional schooling beyond normal school through individual tutoring. While the sixth-grade high-stakes test (summative assessment) is used to determine the nature of the lifetime education an individual receives, the huge pressure that the count-down to this test generates is diffused by the additional schooling after school, or individual tutoring. The significance of the Primary School Leaving Examination (PSLE) and its overall success cannot be isolated from the school after school or individual tutoring program that almost invariably incorporates formative assessment.

Conclusion

An attempt has been made in an eclectic fashion to facilitate a relevant discourse in the promotion of mathematical understanding and proficiency through the assessment lens. The synthesis of formative and summative assessments appears to have arrived at the conclusion and also realized the contention of the fact that their judicious combination would be complementary in the quest for mathematical understanding and proficiency.

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